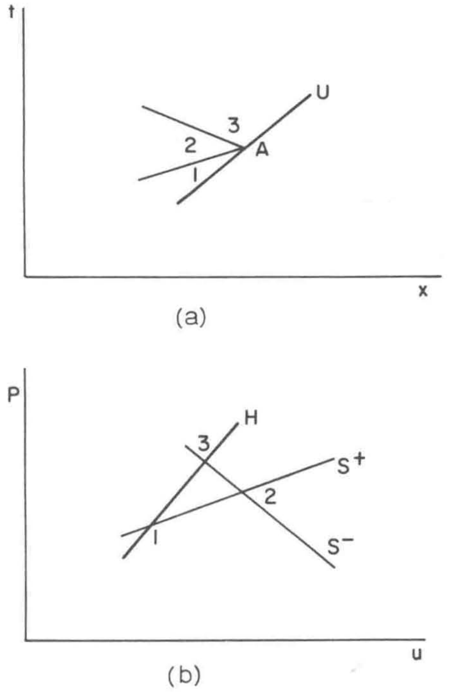

FIG. 8. Reflection of an acoustic wave at a shock front. (a) Time-distance plane. Reflection from shock front at A. (b) Corresponding pressure-particle velocity plane. Numbered states correspond to those at part (a). H is Hugoniot curve, S^+ and S^- are characteristic curves.

1 is the initial shocked state; the state behind the incident acoustic wave, assumed to be a compressional wave, is state 2; and the state behind the reflected acoustic wave is state 3.

The amplitudes of the acoustic waves are assumed to be small; consequently, we retain only first-order terms, and,

$$P_3 - P_1 = (dP/du)_H (u_3 - u_1) + \cdots ,$$

$$P_2 - P_1 = (j/M)(u_2 - u_1) + \cdots ,$$

$$P_3 - P_2 = (-j/M)(u_3 - u_2) + \cdots ,$$

where Eq. (13) has been employed. Eliminating the velocities among these equations yields,

$$u_3 - u_1 - (u_3 - u_2) - (u_2 - u_1)$$

$$= \left(\frac{du}{dP}\right)_H (P_3 - P_1) + \frac{M}{j}(P_3 - P_2) - \frac{M}{j}(P_2 - P_1) = 0 .$$

Or, in obvious notation,

$$\frac{P_{32}}{P_{21}} = \frac{M - j(du/dP)_H}{M + j(du/dP)_H} , \tag{18}$$

is the ratio of amplitudes of the reflected and incident acoustic waves.

As noted the subsonic condition requires

$$0 < M < 1 ; \; j(du/dP)_H < 1 ,$$

and this condition clearly must be satisfied in order that a reflection occur at all. Let us first, therefore, consider a portion of the range within the limits of Ineq. (17), namely,

$$-1 \le j^2(dV/dP)_H \le 1 , \tag{19}$$

or, from Eq. (10),

$$0 \le j(du/dP)_H \le 1 . \tag{20}$$

From Eqs. (18) and (20) we deduce,

$$0 \le j\left(\frac{du}{dP}\right)_H = \frac{M(1 - P_{32}/P_{21})}{1 + P_{32}/P_{21}} \le 1 .$$

This gives

$$-1 \le \frac{M-1}{M+1} \le \frac{P_{32}}{P_{21}} \le 1 , \tag{21}$$

as the only solution. Within the restrictions specified by Ineq. (19) or (20), therefore, the absolute magnitude of the amplitude of the reflected acoustic wave is not greater than that of the incident wave.

The remainder of the region limited by Ineq. (17) is,

$$1 < j^2(dV/dP)_H < 1 + 2M . \tag{22}$$

Using Eq. (10) this can be written

$$-M < j(du/dP)_H < 0 ,$$

whence, we deduce from Eq. (18),

$$1 < P_{32}/P_{21} .$$

We conclude that amplification of acoustic wave amplitudes occurs in the region specified by Ineq. (22). This is just the region for which multi-valued solutions to the impact problem are admitted by Ineq. (17), and this suggests that shocks in this region are at least conditionally unstable.

It has been shown earlier that an oscillatory type of instability can occur under these circumstances.[6] Thus, for example, consider the special case illustrated in Figs. 9 and 10. A shock to state 1 is perturbed by applying a pressure increment at the boundary, $x=0$, at time t_1, and the pressure at the boundary is then held at its new value P_2, indefinitely. This perturbation is transmitted into the shocked region along a C^+ characteristic and undergoes successive reflections

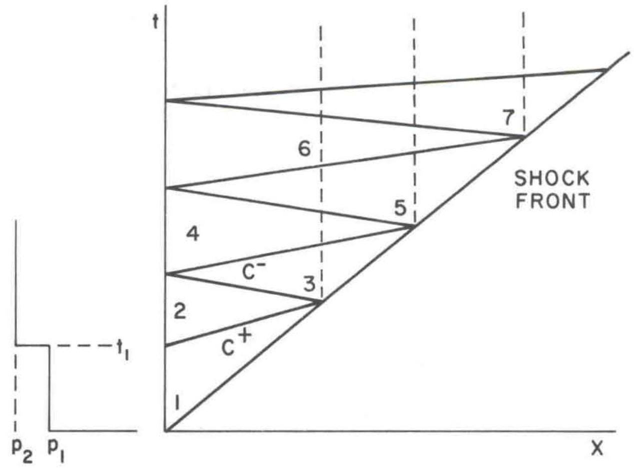

FIG. 9. Time-distance plane showing shock wave and acoustic interactions. Boundary $x=0$ is perturbed at time t_1 by imposing constant pressure increment, $P_2 - P_1$. Forward and backward facing acoustic waves are labeled C^+ and C^-. Motion of boundary, $x=0$, and variations in shock velocity neglected.

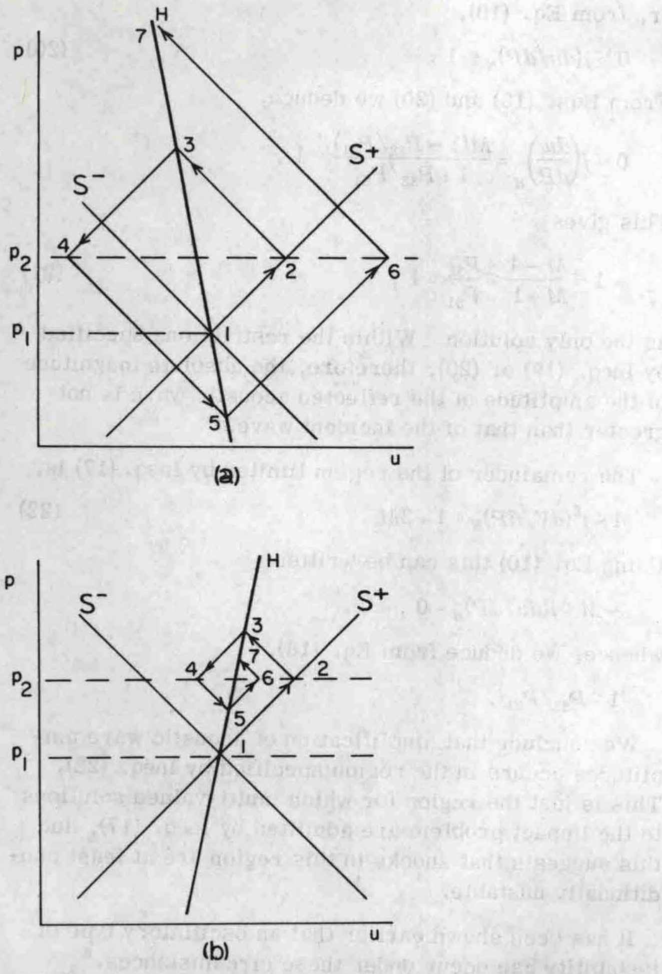

FIG. 10. (a) Pressure, particle velocity plane corresponding to Fig. 9. Numbered states represent P-u states of Fig. 9. Hugoniot, H, has negative slope. Characteristics (isentropes) are labeled S^+ and S^-. (b) Same as Fig. 10(a) except Hugoniot has positive slope.

at the shock front and at the boundary, producing the states labeled 3, 4---. Figure 10 is the associated pressure, particle velocity plane with the numbered states corresponding to those of Fig. 9. The Hugoniot of the material is labeled H and the Γ characteristics, or isentropes, by S^+ and S^-.

The Hugoniot in Fig. 10(a) is assumed to have negative slope and, as a result, the successive reflections form a kind of divergent spiral about the original state, 1. Conversely, it can easily be seen that when the Hugoniot has a positive slope, the spiral is convergent and the state asymptotically approaches a new Hugoniot state at P_2 as in Fig. 10(b).

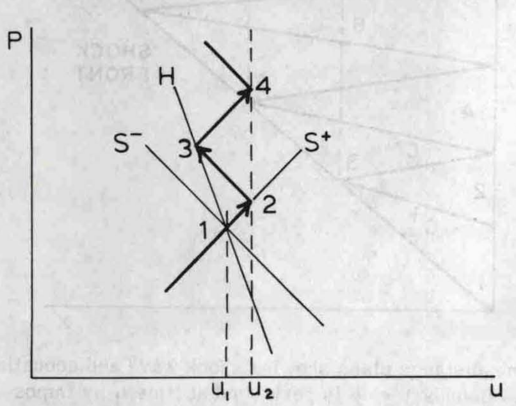

FIG. 11. Similar interaction as shown in Figs. 9 and 10 except perturbation at boundary is in particle velocity, $u_2 - u_1$ = const.

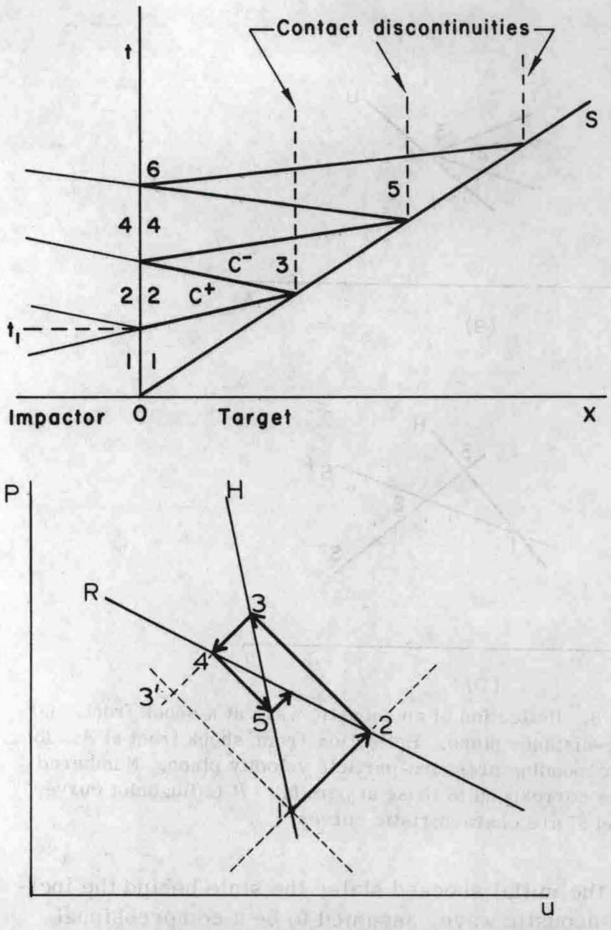

FIG. 12. (a) Similar diagram to that of Fig. 9 except boundary condition is determined by properties of impactor material to left of boundary. (b) Pressure, particle velocity plane corresponding to Fig. 12(a).

Another special case is one in which the perturbation is assumed to be an increment in particle velocity, $u_2 - u_1$, as shown in Fig. 11. When the slope of the Hugoniot is negative, the successive acoustic reflections again grow in amplitude with time as illustrated.

The diagrams of Figs. 9–11 have been simplified in an important respect. Each time an acoustic interaction occurs at the shock front a contact discontinuity is produced, as indicated by the dashed lines in Fig. 9. These present contrasts in acoustic impedance to the acoustic waves with the result that additional internal reflections occur, complicating the process. We know no simple method for treating these internal reflections analytically, but note that they have the ultimate effect of increasing the entropy of the shocked region.

Both of the cases illustrated in Figs. 10 and 11 have a common feature: no acoustic energy is transmitted across the boundary at $x = 0$. If we consider a more general case in which the shock is produced by impact